From WCCS (Wisconsin County Coordinate System) To WISCRS (Wisconsin County Reference Systems)

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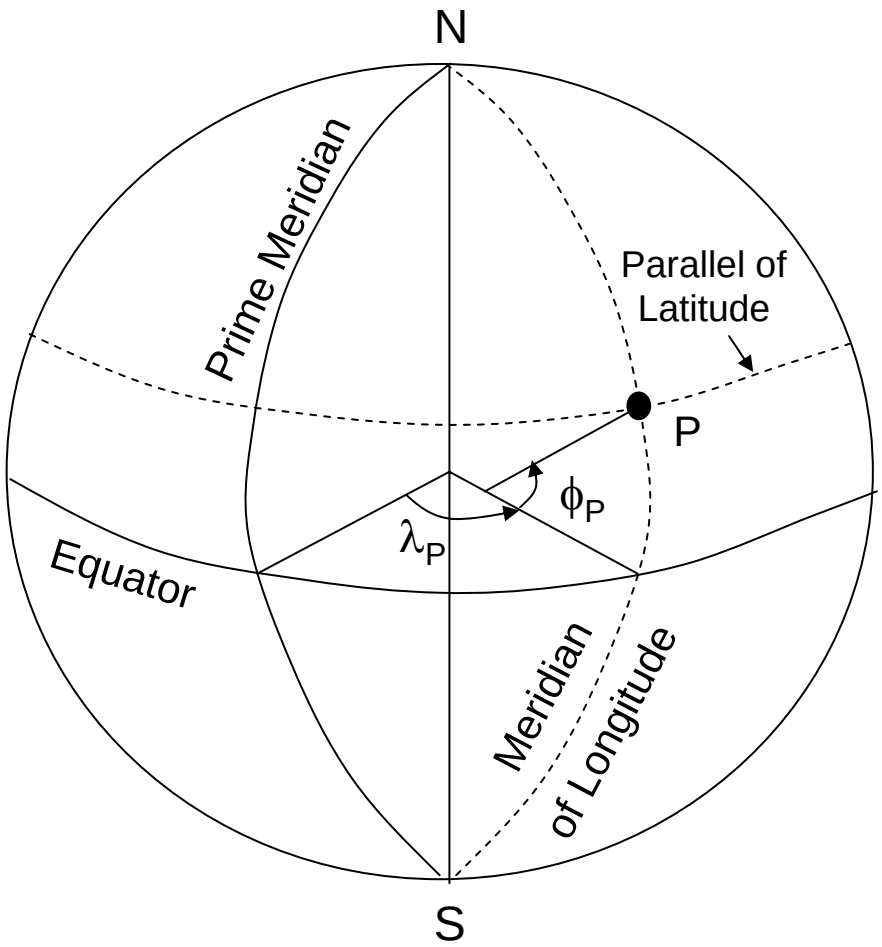
SIAC Seminar – February 15, 2006

Fundamental Descriptors of Position



Ellipsoid

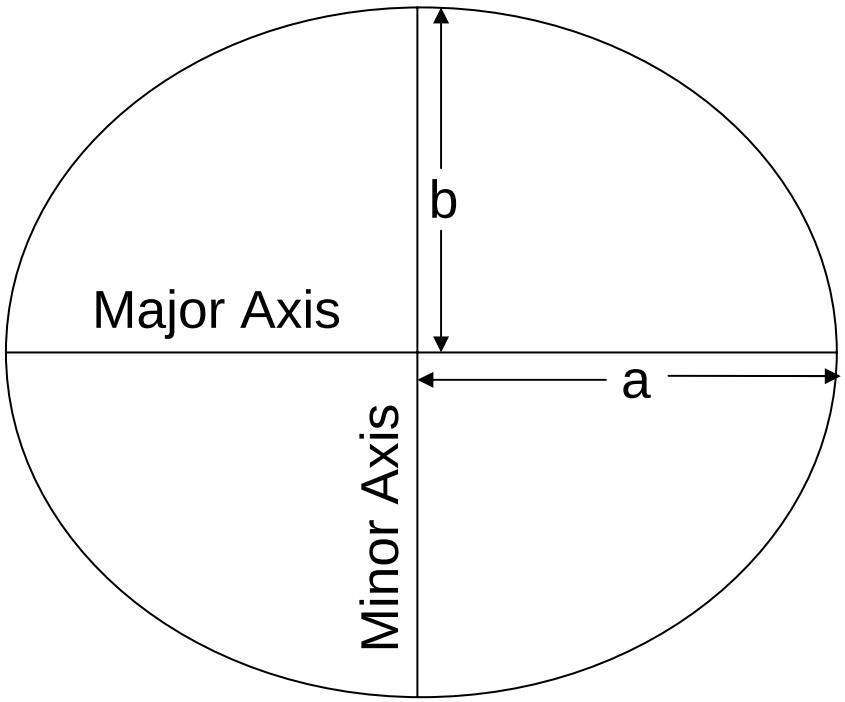
Meridian of Longitude

Prime Meridian

Parallel of Latitude

Latitude (ϕ_P) and Longitude (λ_P) of point P

Elements of an Ellipse



a = Semi-Major Axis

b = Semi-Minor Axis

Rotate about minor axis to generate oblate spheroid.

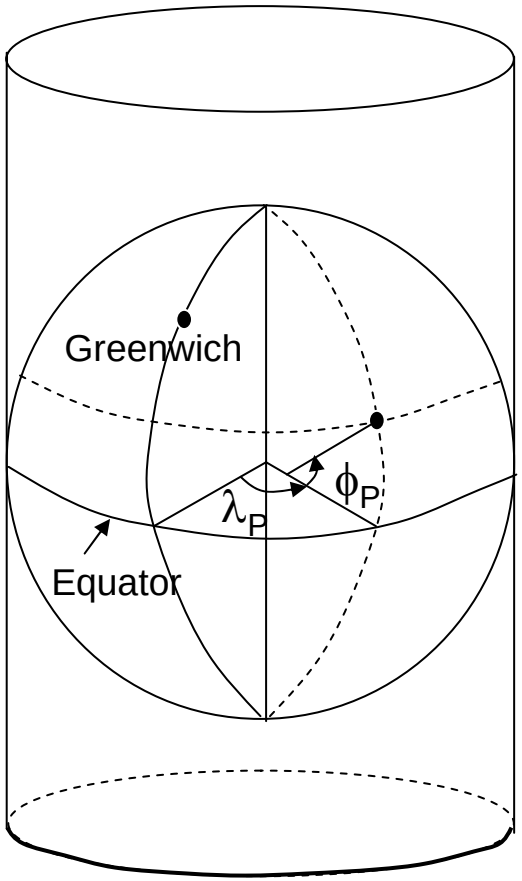
Spheroid used for current national geodetic datum (NAD83) is named “GRS 80”:

- a = 6378137.0 m
- b = 6356752.3141403 m

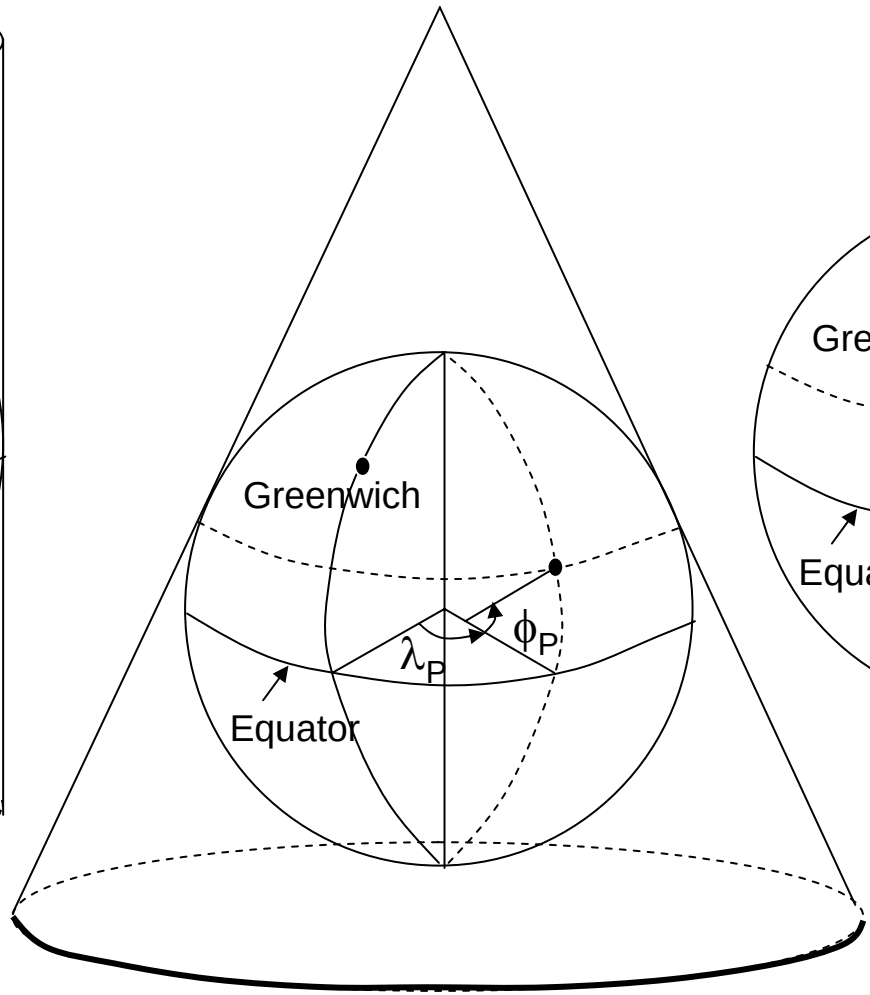
Computational and Visualization Problem

- Latitude / Longitude are angular, not rectangular coordinates.
- Ellipsoid surface cannot be cut and laid flat.
- Latitude / Longitude must be projected to a “developable” surface to obtain rectangular coordinates.

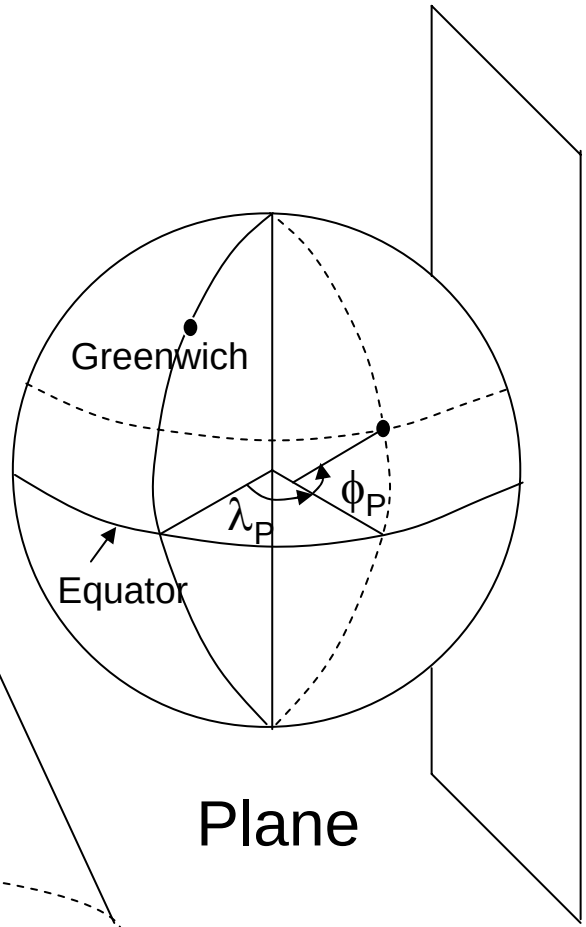
Developable Surfaces



Cylinder

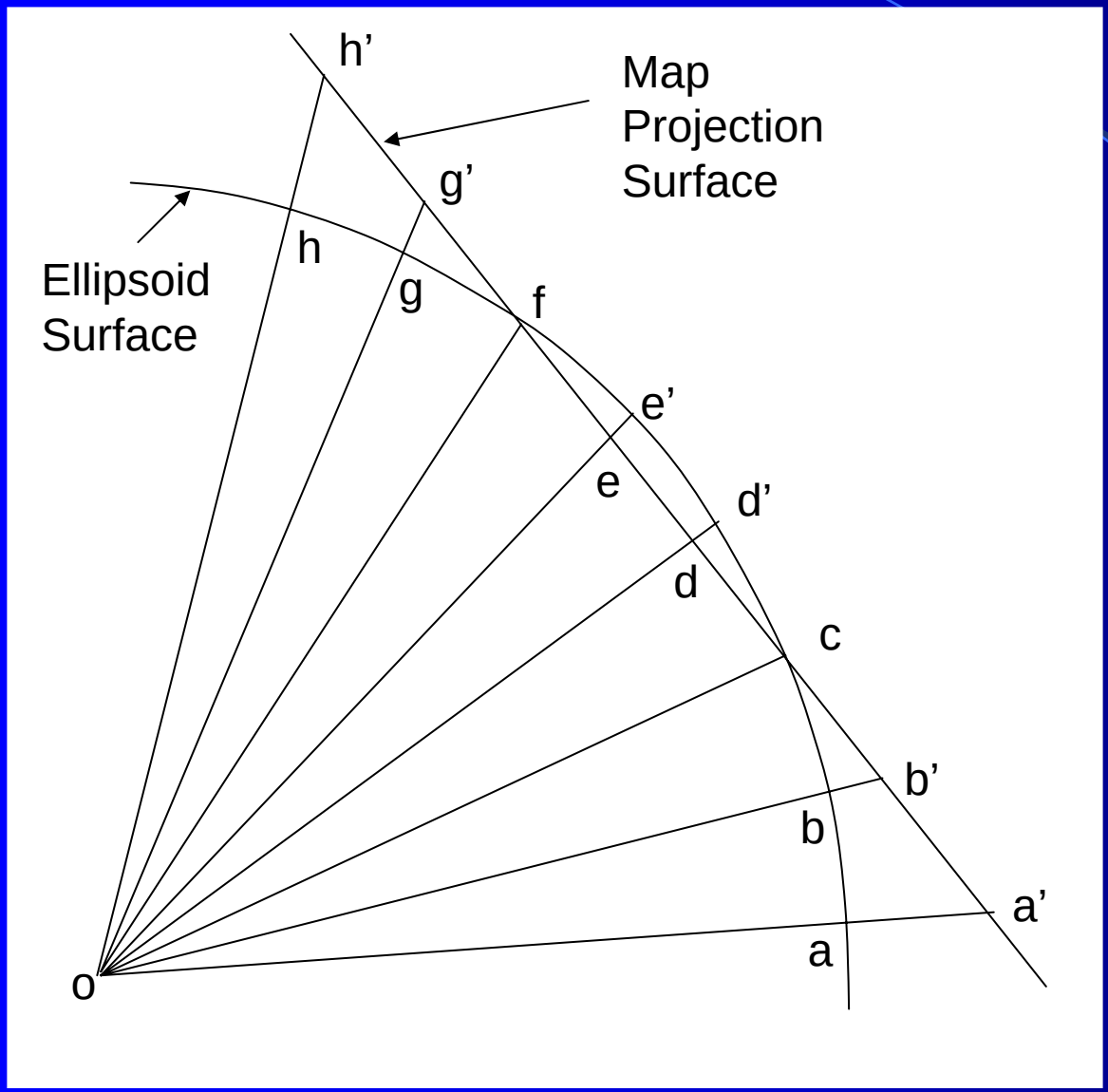


Cone



Plane

One Way to Conceptualize “Projection”



Points on the ellipsoid are projected to the projection surface by straight lines from the center of the ellipsoid.

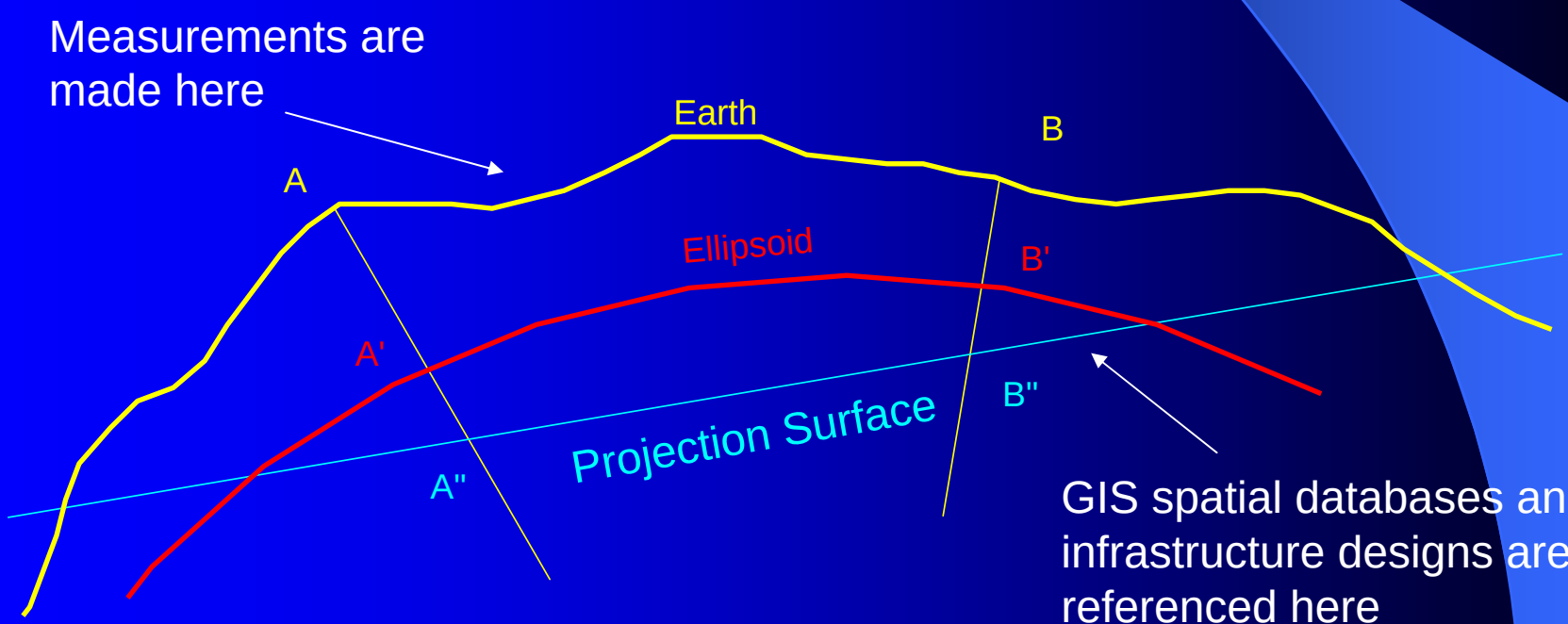
Note scale factor and how it varies across the projection surface.

Note: Some map projections are purely mathematical and have no graphical counterpart.

Ground-to-Grid

Problem: Length distortion occurs when projecting from:

- Ground (Earth) to ellipsoid
- Ellipsoid to projection surface



Ground-to-Grid

- Two step process to obtain grid (map projection) distances from ground distances:

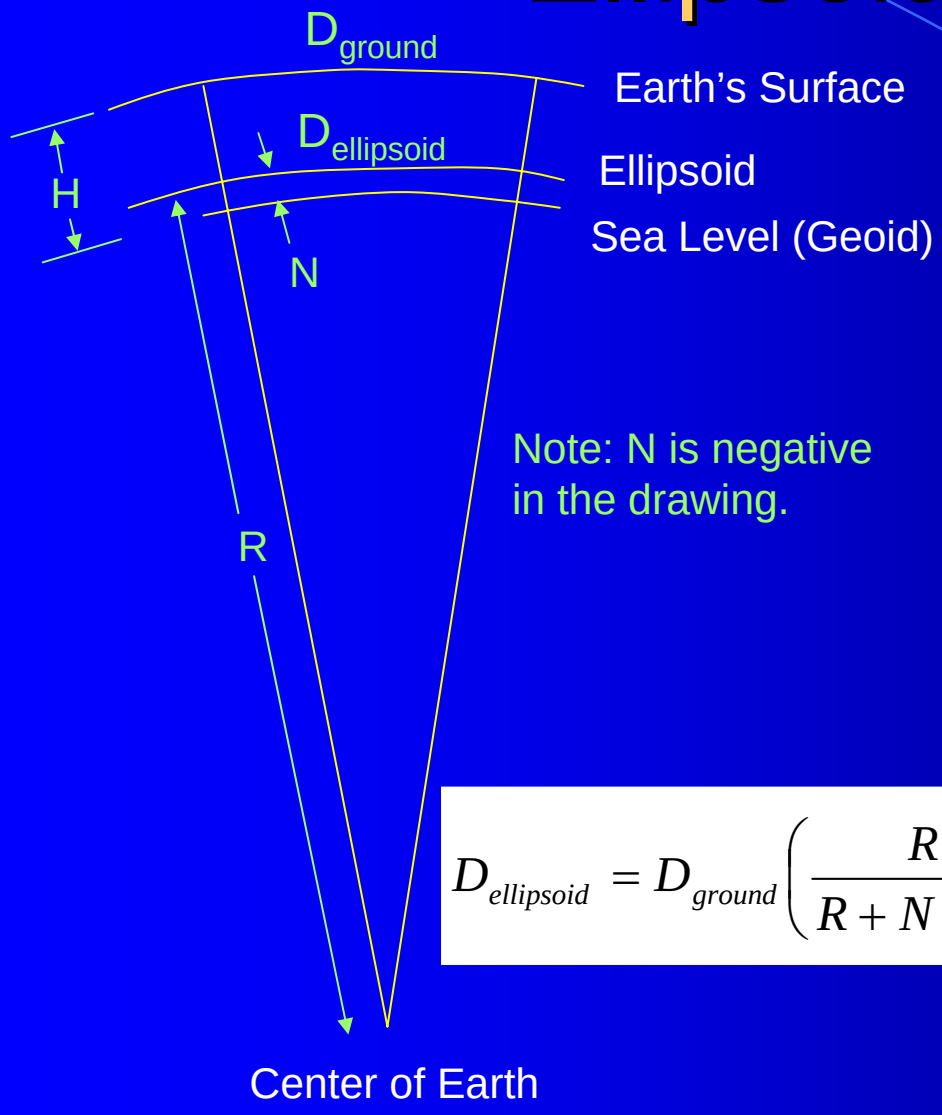
$$D_{ellipsoid} = (D_{ground})(EllipsoidFactor)$$

$$D_{grid} = (D_{ellipsoid})(ScaleFactor)$$

- Or

$$D_{grid} = (D_{ground})(EllipsoidFactor)(ScaleFactor)$$

Ellipsoid Factor



Note: N is negative in the drawing.

R / (R + N + H) is called the "ellipsoid factor".

R is computed from a, b, and ϕ.

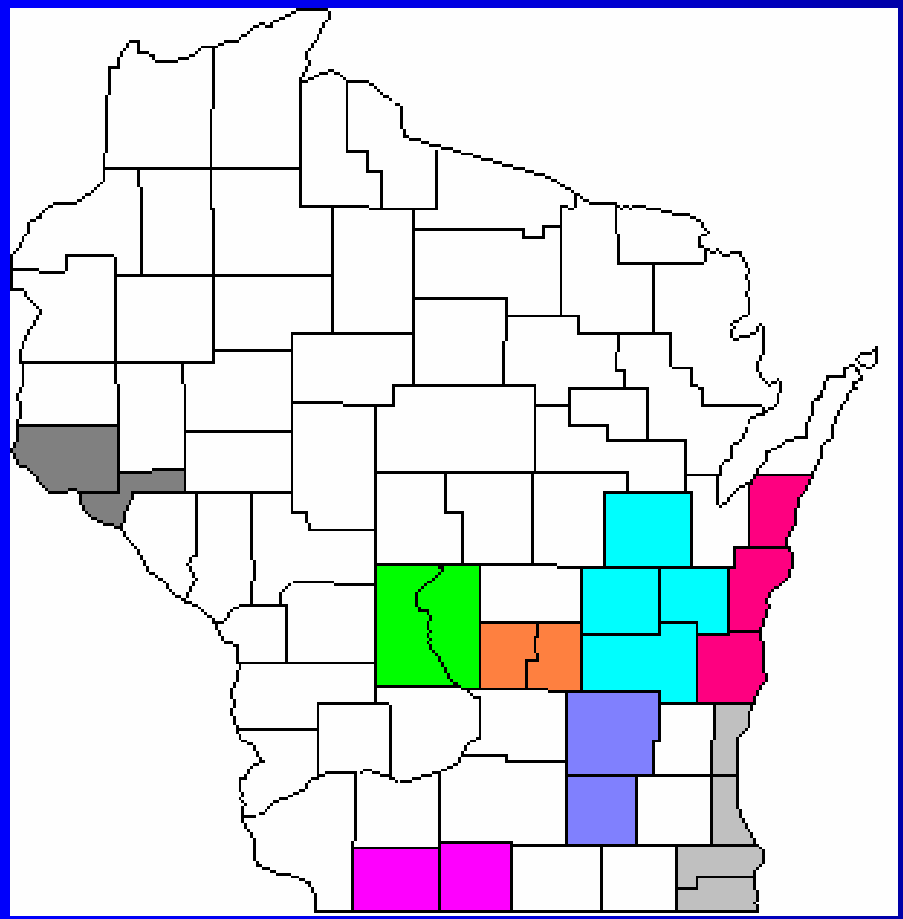
Ellipsoid factor varies with position.

$$D_{ellipsoid} = D_{ground}\left(\frac{R}{R+N+H}\right)$$

Wisconsin County Coordinates

- Original WCCS Objective:
  1. Make differences between ground distances and grid distances negligible for most applications.
- Original Design Strategy:
  1. Restrict extent of each projection so scale factor is approximately equal to one everywhere.
  2. For each projection, enlarge the ellipsoid by adding N+H to R. This causes the ellipsoid factor to be approximately equal to one everywhere.

Wisconsin County Coordinates



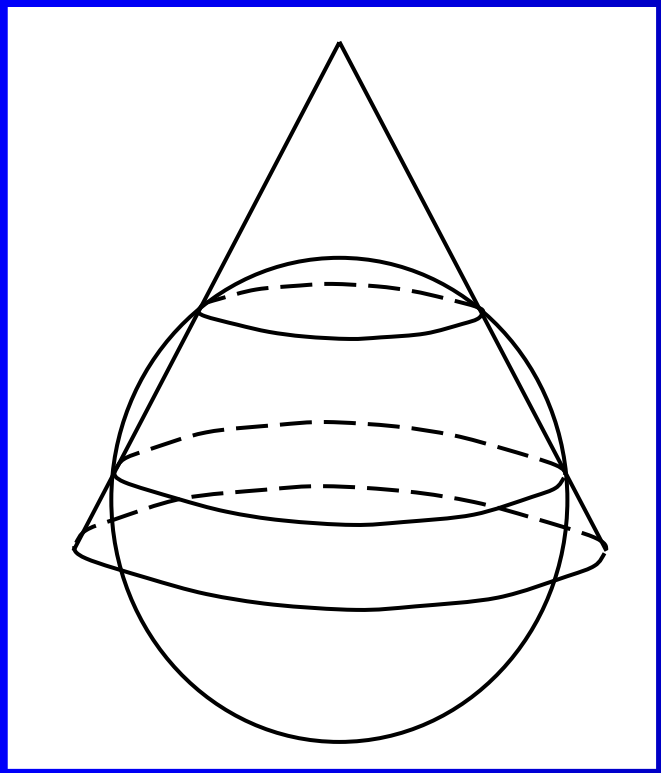
72 Counties

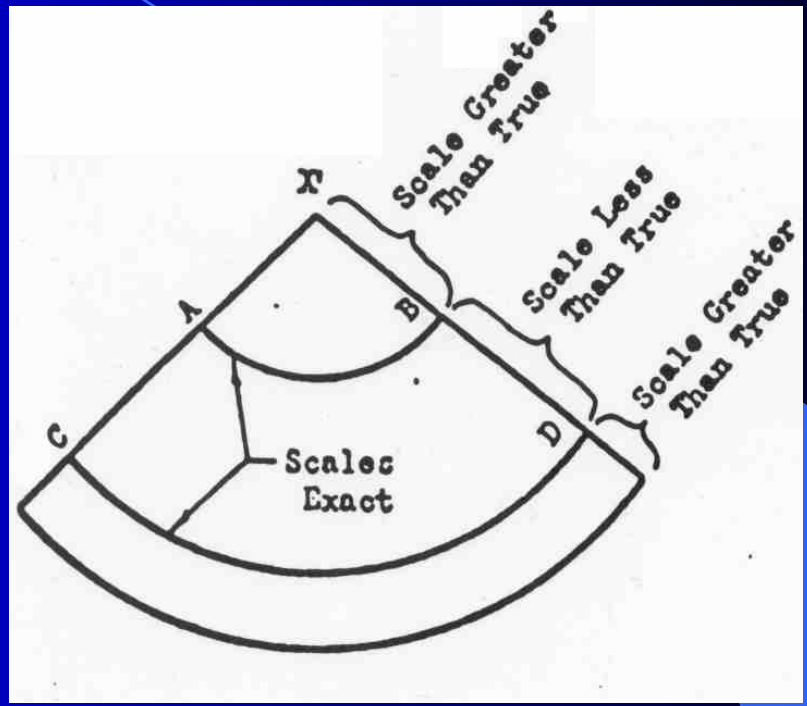
59 Coordinate Systems

24 Lambert

35 Transverse Mercator

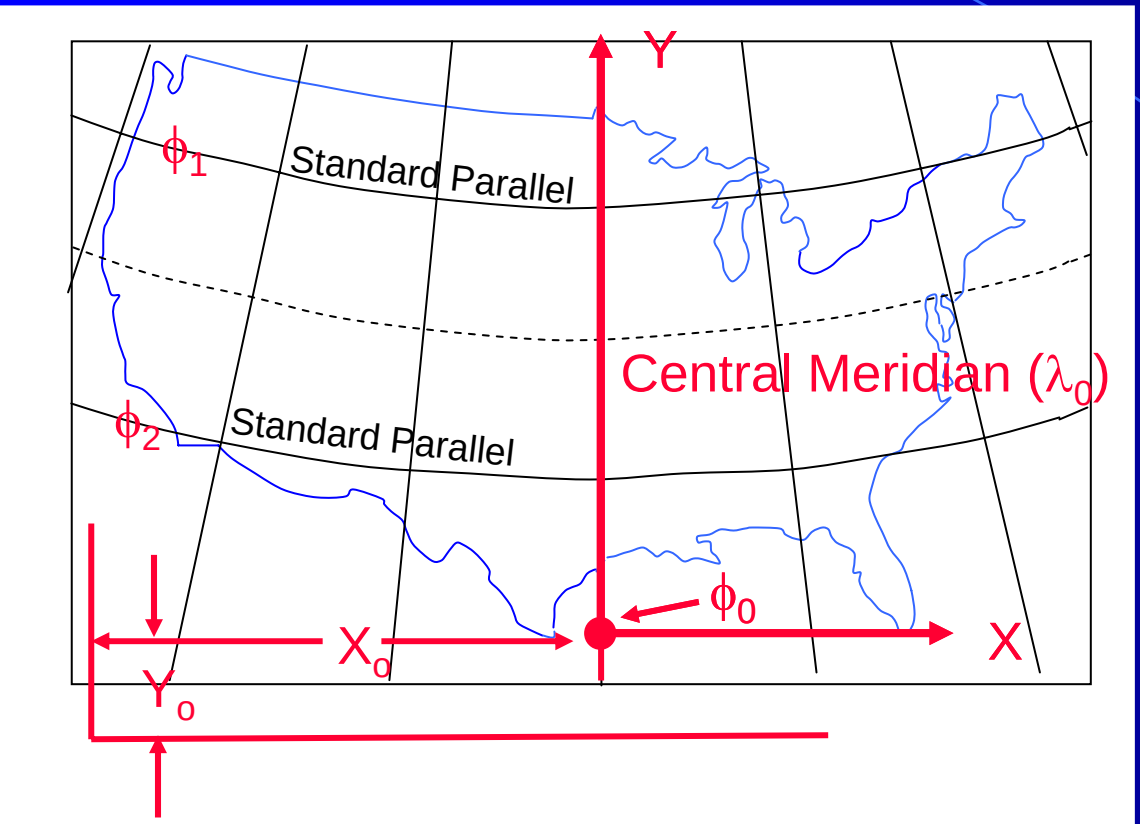
Lambert Conformal Conical Projection





Scale variation is greater north-south than east-west.

Lambert Conformal Conical Projection



Projection Parameters:

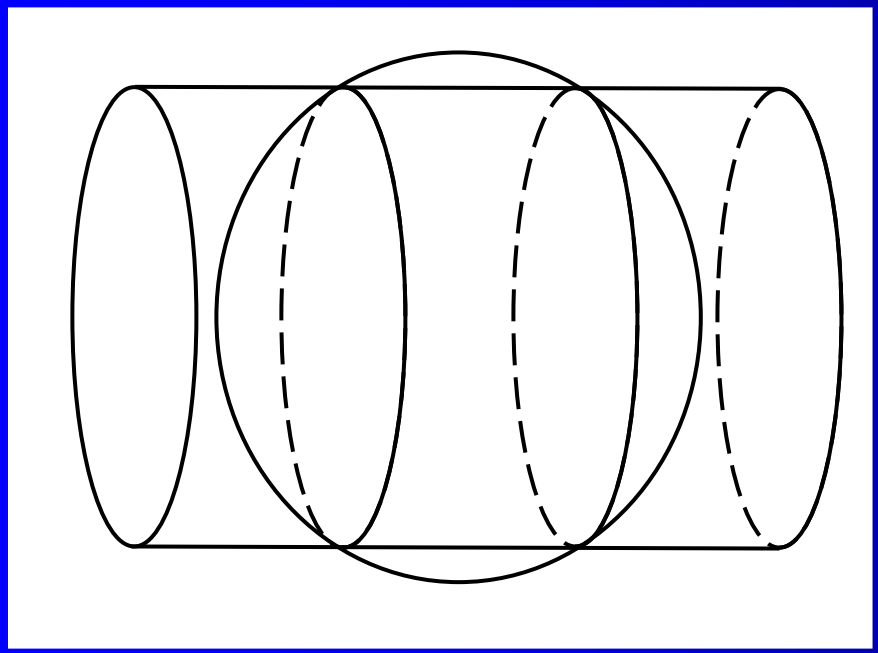
λ_0 (longitude of central meridian)

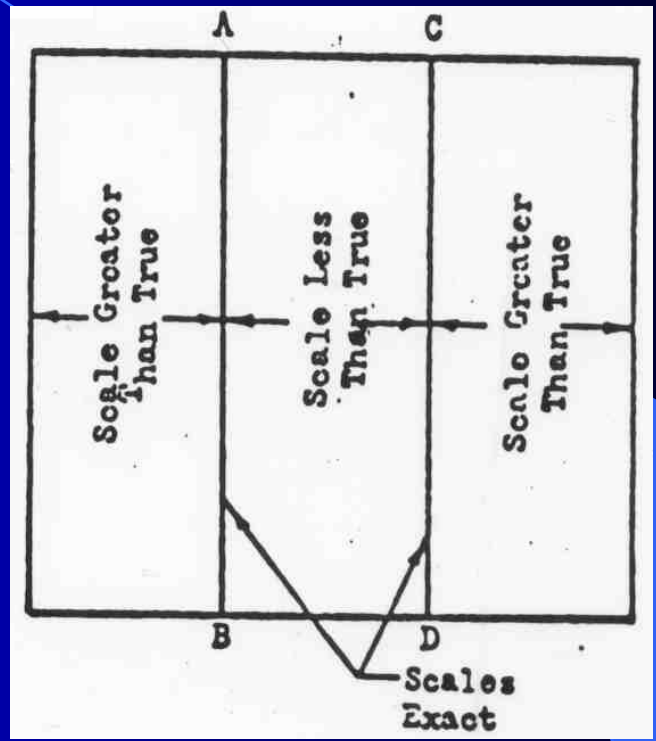
ϕ_1, ϕ_2 (latitudes of standard parallels)

ϕ_0, X_0, Y_0 (latitude, false easting, false northing of the coordinate origin)

Alternative to ϕ_1, ϕ_2 is ϕ_0,k_0 (latitude and scale factor at central parallel).

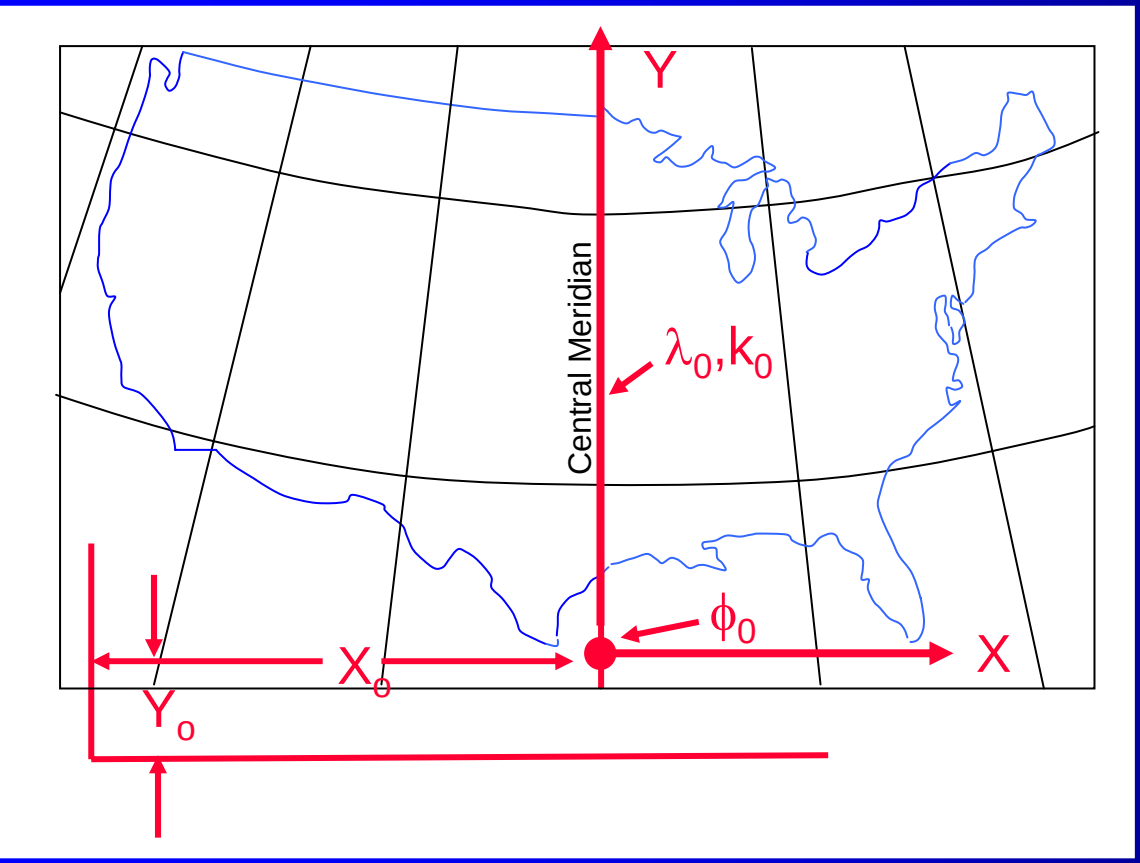
Transverse Mercator Projection





Scale variation is greater east-west than north-south.

Transverse Mercator Projection



Projection Parameters:

λ_0 (longitude of central meridian)

k_0 (scale factor along central meridian)

ϕ_0, X_0,Y_0 (latitude, false easting, false northing of the coordinate origin)

Wisconsin County Coordinates

- Problem:
  - Each projection has its own ellipsoid.
  - This makes it seem like each projection has its own datum.
  - Confusion abounds.

WLIA Task Force

- In 2004, WLIA formed the Wisconsin Coordinate Systems Task Force to address this and other spatial referencing issues.
- Ultimately, the Task Force recommended redesign of the system, established criteria, and obtained funding.

Redesign Objectives

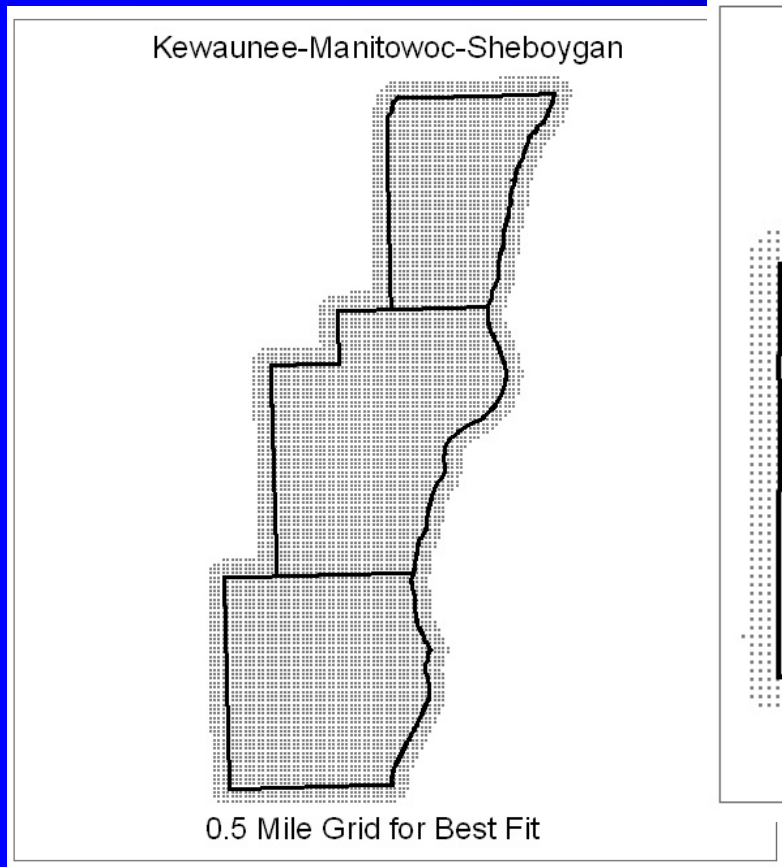
1. Redesign the coordinate systems so there is no need to enlarge the ellipsoid.
   - There will be only one ellipsoid (GRS80) for everyone.
2. Redesigned coordinates should not differ by more than 5mm from the originals anywhere on any projection.
   - Legacy data will be preserved.
   - Existing and new data can be combined without transforming either.

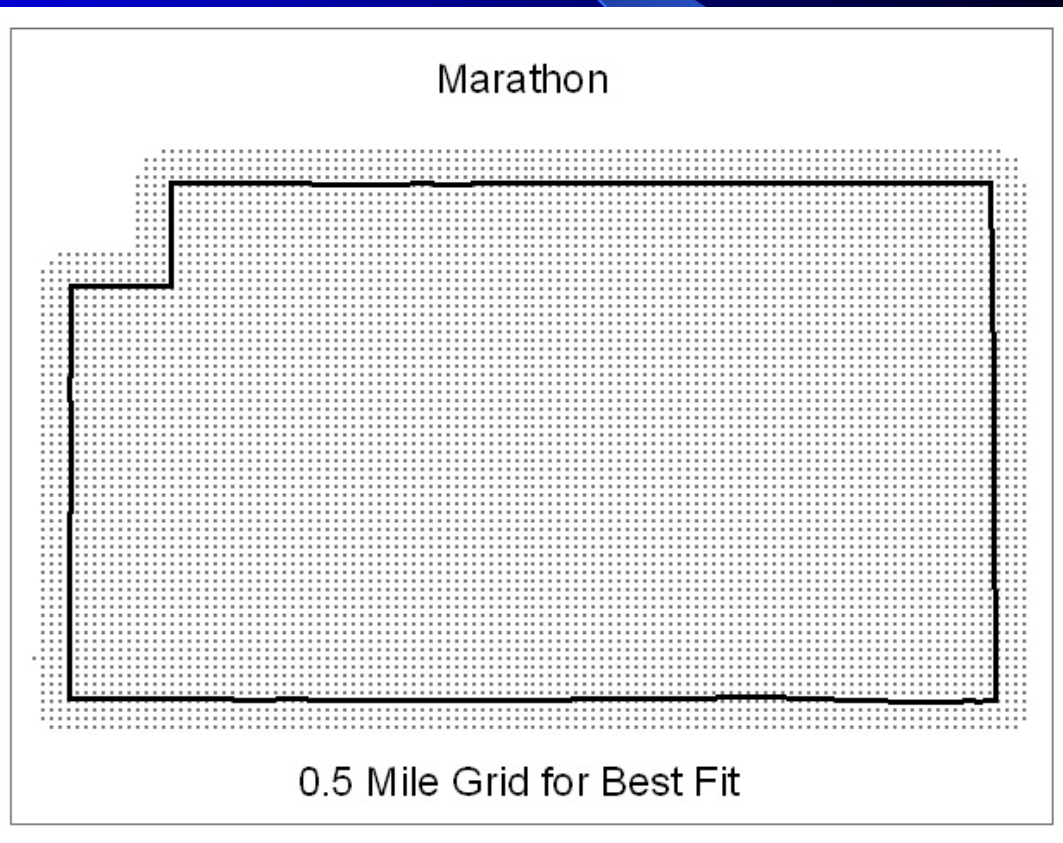
Redesign Strategy

1. Multiply scale factor on Central Meridian (Transverse Mercator) or Central Parallel (Lambert) by (R + N + H) / R to obtain provisional scale factor.
   - Causes ellipsoid factor and scale factor to be approximate reciprocals of one another, so when they are multiplied together the result is approximately equal to one.
2. Adjust false northing, false easting, and provisional scale factor to account for effects of difference in eccentricities of the two ellipsoids (GRS80 and enlarged).

Redesign Methodology

- Methodology:
  1. Use DNR statewide map to obtain boundaries for each projection.
  2. Generate a 0.5-mile grid of test points within a 2-mile buffer for each projection.





Redesign Methodology

- Methodology:
  3. Compute provisional scale factor for each projection.
  4. Using provisional scale factor, compute provisional county coordinates for each grid point.
  5. Compute original county coordinates for each grid point.
  6. Develop observation equations for each grid point:

$$(E_{original} - E_o) + v_{Eoriginal} = S(E_{provisional} - E_o) + \Delta E_o$$

$$(N_{original} - N_o) + v_{Noriginal} = S(N_{provisional} - N_o) + \Delta N_o$$

Redesign Methodology

- Methodology:
  7. Compute least squares solution of about 10,000 equations for each projection to obtain shifts in false northing and false easting, and multiplier for provisional scale factor.
  8. Final Transverse Mercator parameters are:

$$\lambda_{o(redesigned)} = \lambda_{o(original)}; \phi_{o(redesigned)} = \phi_{o(original)}; k_{o(redesigned)} = k_{o(provisional)} * S;$$

$$E_{o(redesigned)} = E_{o(original)} + \Delta E_o; N_{o(redesigned)} = N_{o(original)} + \Delta N_o$$

Number of Transverse Mercator parameters is reduced from 7 to 5.

Redesign Methodology

- Methodology:
  9. Final Lambert parameters are:

$$\lambda_{o(redesigned)} = \lambda_{o(original)}; \phi_{o(redesigned)} = \phi_{o(original)}; k_{o(redesigned)} = k_{o(provisional)} * S;$$
$$E_{o(redesigned)} = E_{o(original)} + \Delta E_o; N_{o(redesigned)} = N_{o(original)} + \Delta N_o$$

  - Number of Lambert parameters is reduced from 8 to 5.
  - $\phi_{o(original)}$ is computed from $\phi_{1(original)}$ and $\phi_{2(original)}$.
  - Coordinate origin is shifted to ϕ_o, λ_o.
  - $N_{o(original)}$ at new coordinate origin is computed, not given.

Redesign Methodology

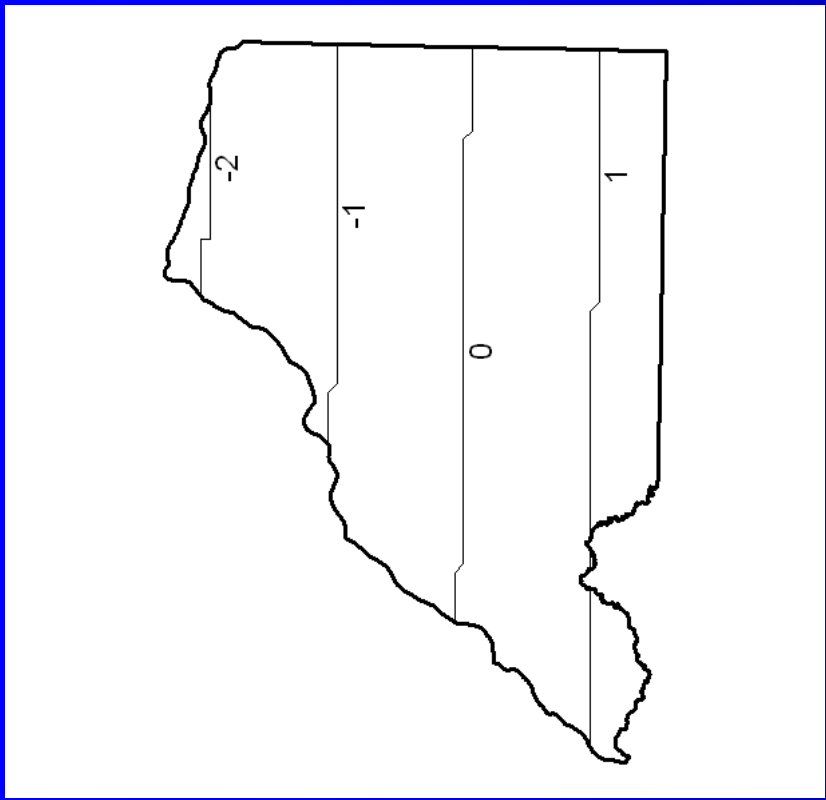
- Methodology:
  10. Compute differences between redesigned and original coordinates at each grid point.
  11. Find maximum shifts in northings and eastings to check against 5mm tolerance.
  12. Prepare isoline (contour) maps of coordinate shifts.

Redesign Results

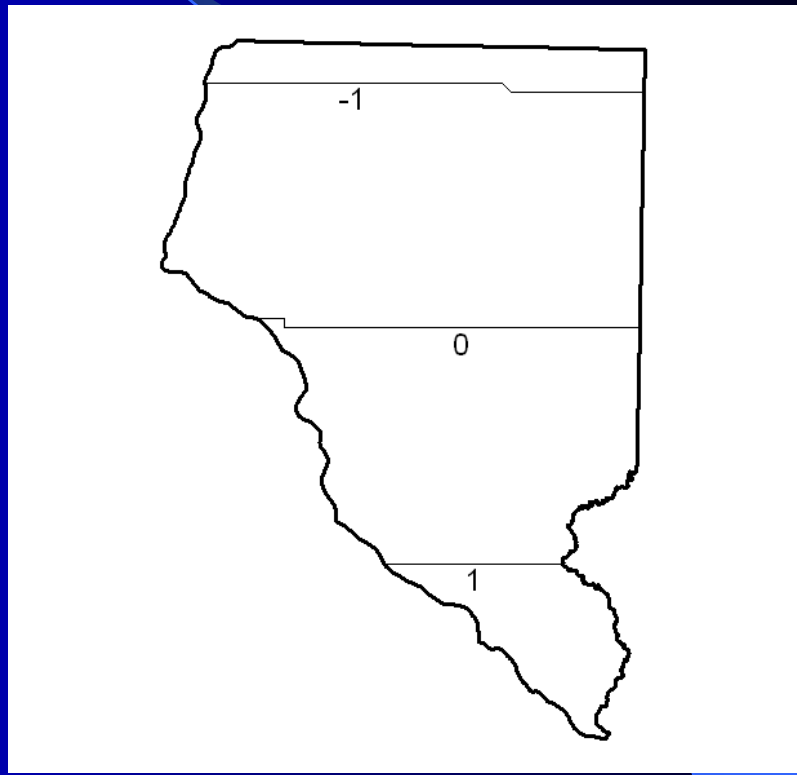
- Results:
  - All coordinate systems meet the redesign criterion:
    - All coordinate shifts are less than 5mm.
  - Typical coordinate shifts range from –3mm to +3mm.
  - Some counties have maximum shifts of less than 1mm.
  - Maximum shifts are in Oneida and Vilas (Lambert) and Ashland and Forest (Transverse Mercator).

Coordinate Shifts

Buffalo County (Typical Transverse Mercator)



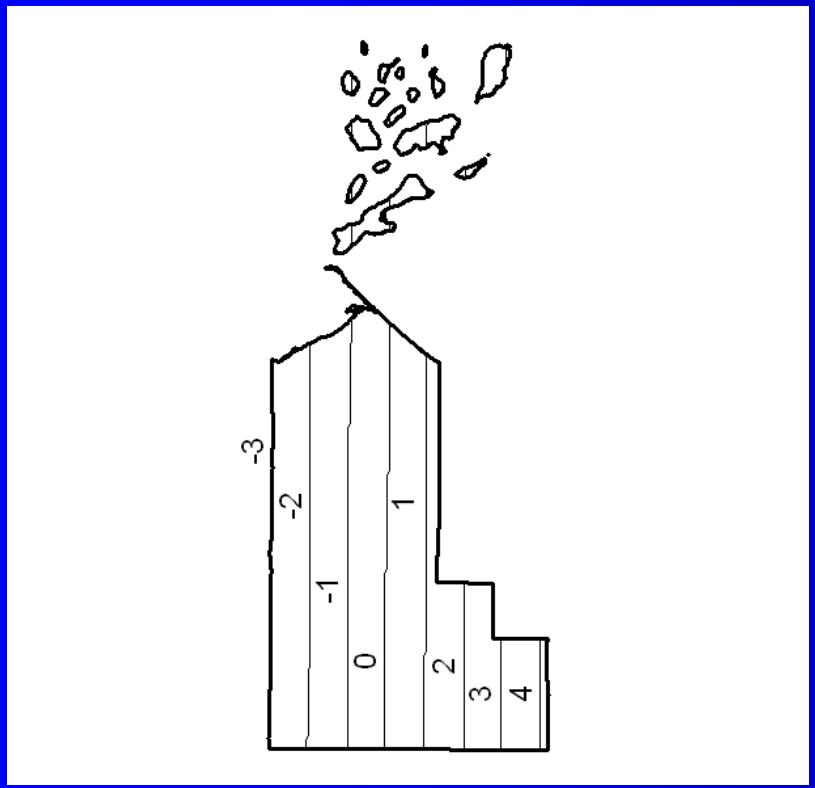
Shift in Easting (mm)



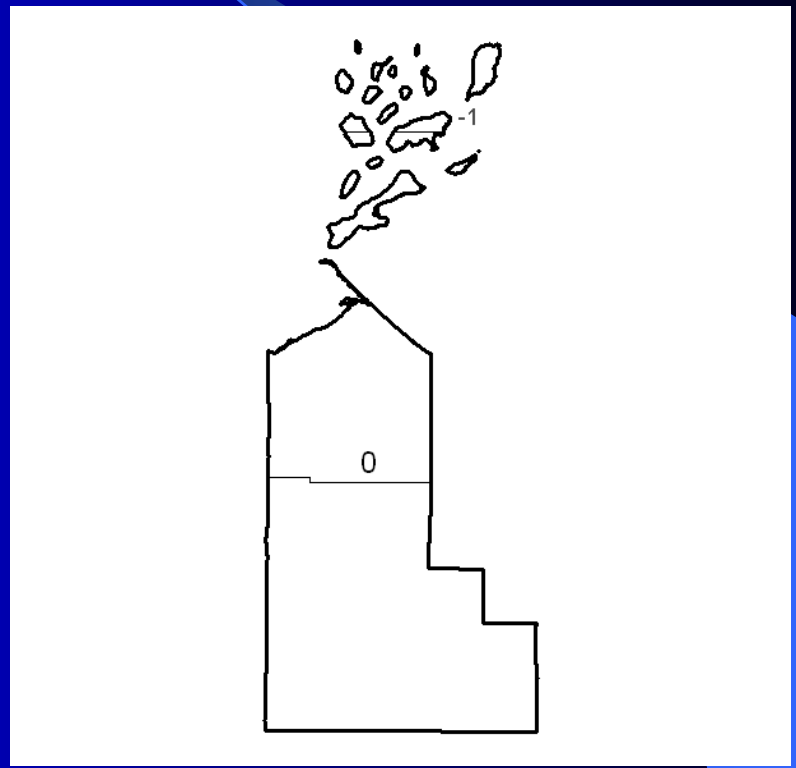
Shift in Northing (mm)

Coordinate Shifts

Ashland County (Worst-Case Transverse Mercator)



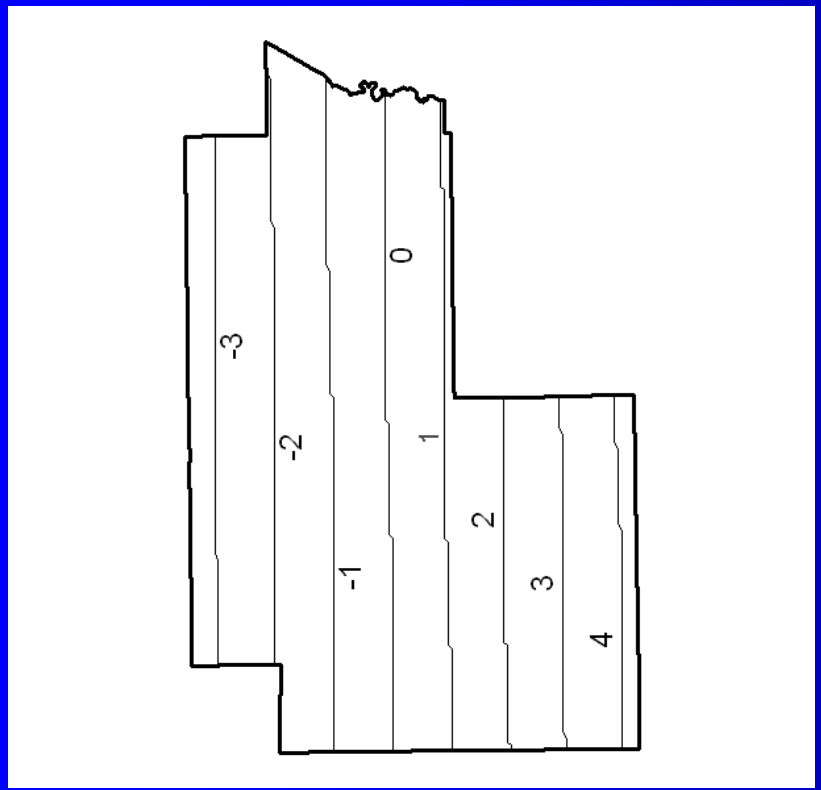
Shift in Easting (mm)



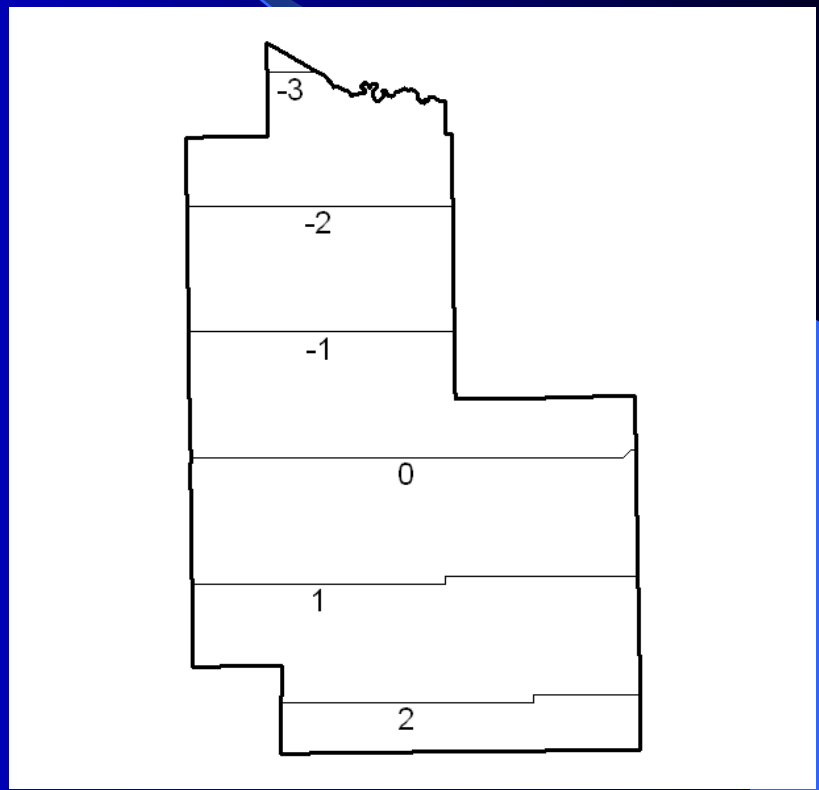
Shift in Northing (mm)

Coordinate Shifts

Forest County (Worst-Case Transverse Mercator)



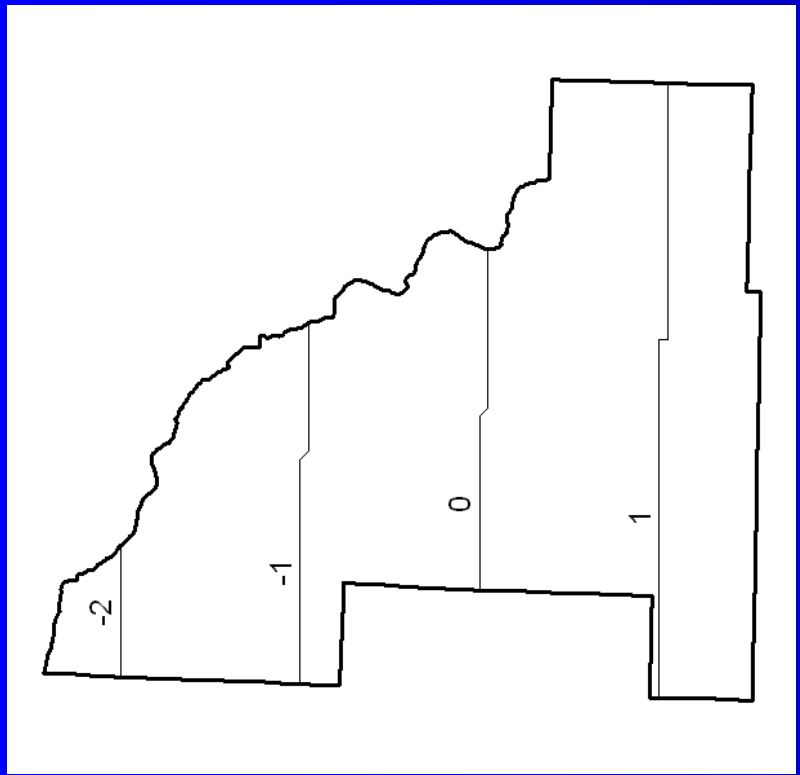
Shift in Easting (mm)



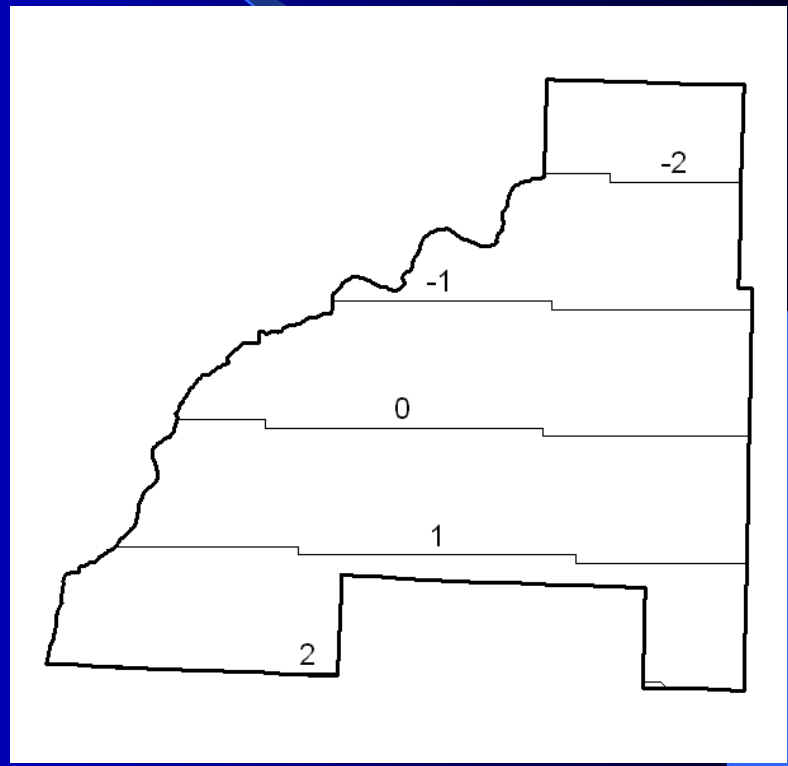
Shift in Northing (mm)

Coordinate Shifts

Burnett County (Typical Lambert)



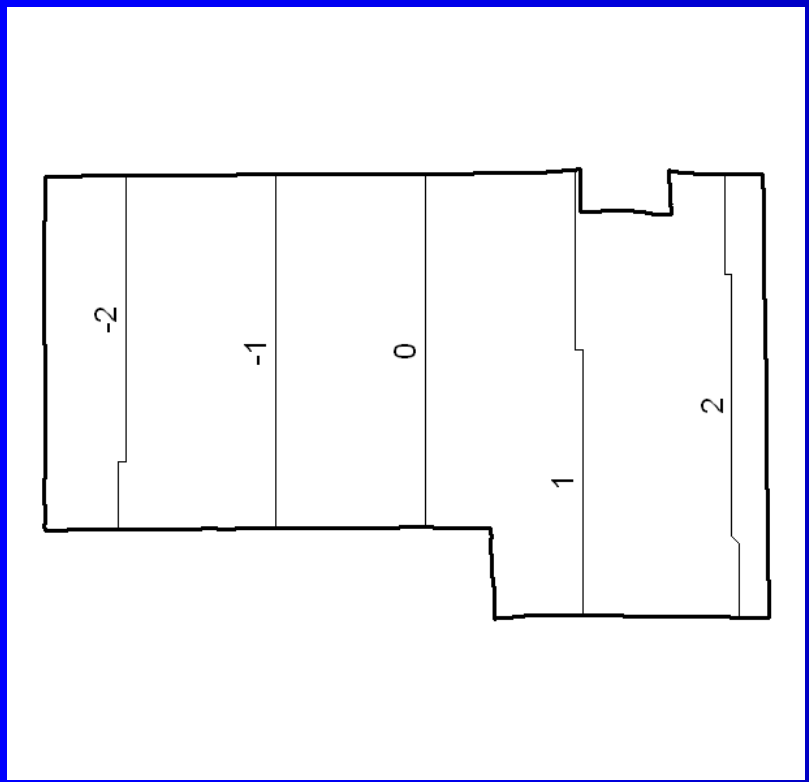
Shift in Easting (mm)



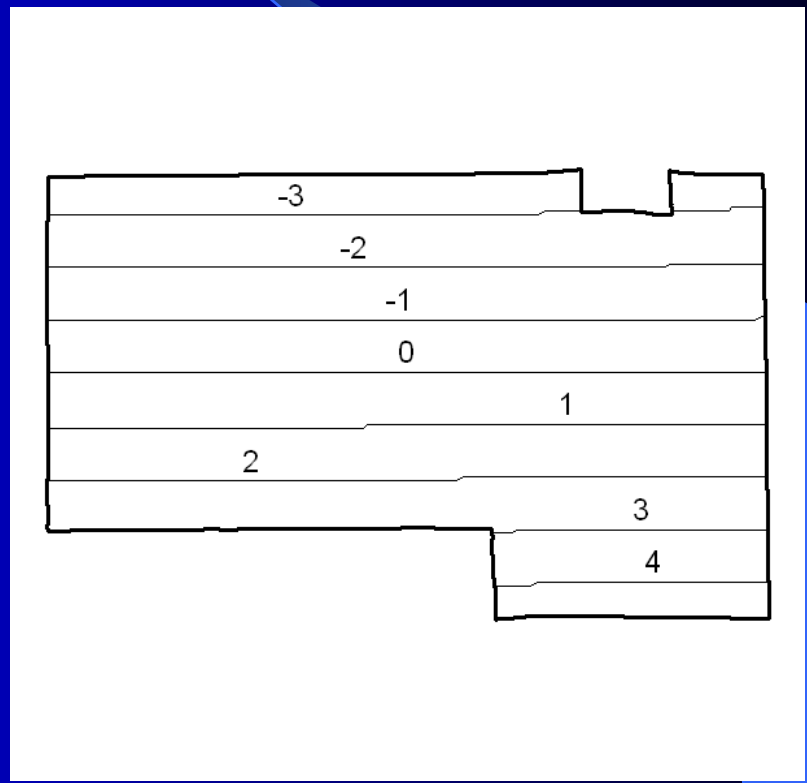
Shift in Northing (mm)

Coordinate Shifts

Oneida County (Worst-Case Lambert)



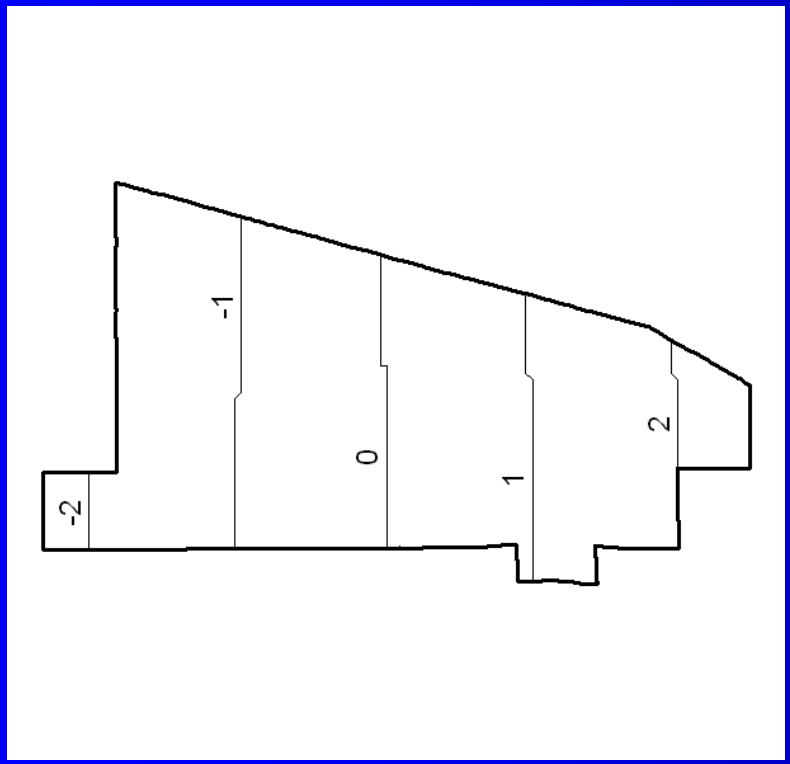
Shift in Easting (mm)



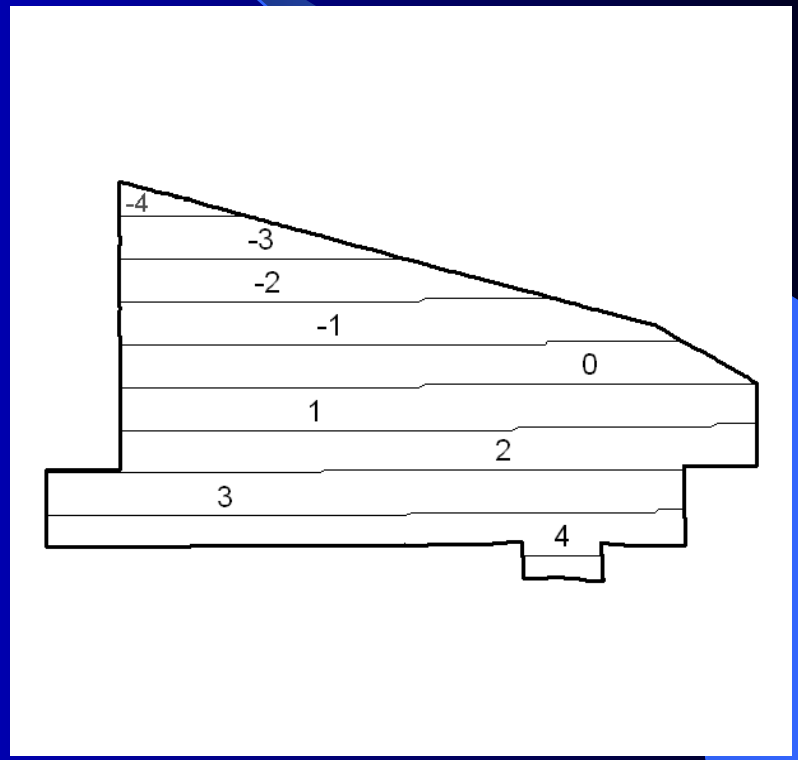
Shift in Northing (mm)

Coordinate Shifts

Vilas County (Worst-Case Lambert)



Shift in Easting (mm)



Shift in Northing (mm)

Status

- Validation:
  - Independent testing by four individuals using various software packages and programming techniques.
  - All have concluded that the redesign meets the 5mm criterion.
- Draft final report under review.
  - Final submittal during March.

WISCRS (Wisconsin County Reference Systems)

- Two days ago, the Task Force decided:
  - To retain the name “WCCS (Wisconsin County Coordinate System)” for the original.
  - To name the redesigned “WISCRS (Wisconsin County Reference Systems).
    - Individual county systems are suggested to be referred to as “WISCRS, Dane County”, for example.

WISCRS (Wisconsin County Reference Systems)

- It is expected that WISCRS will
  - Be more easily understood by the user community.
  - Be easily adoptable by vendors.
  - Allow integration with legacy data without the need for transformation.
  - Be placed in at least one “official” registry.